



## RESEARCH ARTICLE

**Exotic cluster decay in nuclei with  $Z = 52-62$** **D.R. Roshlin Sheeba Rani<sup>a</sup> and V. Selvam<sup>b</sup> \***

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**Abstract**

The exotic cluster radioactive decay of nuclei with  $Z = 52-62$  are studied using the unified fission model. The half lifetime values are evaluated and then compared with the values obtained by Poenaru et al. who used the analytical super asymmetric fission model. The half-lives obtained for the considered decays lie very close to those by Poenaru et al. The linear nature of Geiger – Nuttal plots observed in the present study show that the effect of proximity potential is mild. The rate of radioactive decay is inversely proportional to neutron excess in the present nucleus is also reported. It is concluded that the presence of neutron excess in the parent nuclei slows down the exotic decay process.

**Keywords**

Exotic decay  
Cluster radioactivity  
Unified fission model  
Proximity potential

**Introduction**

Recent advances in the field of radioactivity are the production of radioactive nuclear beams of neutron rich nuclei and the decay of nuclei with products heavier than the alpha particle but lighter than the so far observed binary fission products. The clusters usually emitted in this process are the alpha particle, carbon, oxygen, neon, magnesium, silicon etc. When the mass of the cluster becomes comparable with the mass of the daughter, symmetric fission takes place. Thus the cluster radioactivity is an intermediate process between the well known alpha decay and the spontaneous fission. This new radioactivity called

cluster radioactivity, which was first predicted theoretically by Sandulescu, Poenaru and Greiner [1] in 1980. In 1984, Rose and Jones [2] experimentally confirmed this prediction and measured the half life times for  $^{14}\text{C}$  emission from  $^{223}\text{Ra}$ . This has triggered enormous activity in the field of Cluster radioactivity both experimentally and theoretically.

To study the phenomenon of cluster radioactivity there are various theoretical models in vogue. The existing models generally fall into two categories: the Unified Fission Model (UFM) and the Preformed Cluster Model (PCM). The UFM considers cluster radioactivity simply as a barrier penetration phenomenon in between the fission and the alpha decay without worrying about the cluster being or not being preformed in the parent nucleus. In the PCM, clusters are assumed to be pre-born in a parent nucleus before they could penetrate the potential barrier with a given  $Q$ - value. The basic assumption of the UFM is

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that heavy clusters as well as the  $\alpha$ - particle have equal probability of being preformed. In PCM, clusters of different sizes have different probabilities of being preformed in the parent nucleus.

Poenaru et al. [3] have developed an Analytical Super Asymmetric Fission Model (ASAFM). This model is a Unified Fission Model and is able to account for three types of radioactive decays namely heavy particle radioactivity,  $\alpha$ - particle decay and cold fission. This model is an improved version of a fission model for alpha decay adapted to heavy cluster decays. This model has been used [4] to calculate half lifetime for proton-rich parent nuclei with  $Z = 56-72$  which decay by  ${}^4\text{He}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$  and  ${}^{28}\text{Si}$  emissions. This region is quite interesting because the daughter nuclei in such decays are formed around the doubly magic  ${}^{100}\text{Sn}$  and the estimated branching ratios are favourable for measurements. Moreover only  $N=Z$  clusters are emitted and  $Z/A$  values for parent, daughter and emitted cluster are nearly equal to 0.5. Experiment for producing such parent exotic nuclei were conducted at Dubna, Russia [5] and at GSI, Darmstadt, Germany [6,7].

In this work we have used the Cubic Potential model (CPM) which uses Yukawa plus exponential potential [8] along with the Coulomb potential for the outer region and a third order polynomial for the overlapping region. The zero point energy is explicitly included in the formalism and also the centre of mass dependent inertial mass coefficient has been used. The height of the barrier is accurately determined. The aim of this work is to study the exotic decay of nuclei in the p- block, s- block and 4f- block elements. The nuclei chosen for the study are Tellurium and Iodine in p- block, Cesium and Barium in s- block and Cerium, Neodymium and Samarium in the 4f- block. In the first phase of this work we have generated the exotic cluster decay lifetimes for various cluster emissions from the above said elements using the unified fission model. The obtained lifetime values are then compared with the values obtained by Poenaru et al. who used the Analytical Super Asymmetric Fission Model (ASAFM) [8]. In the second phase of this work the Geiger - Nuttal plots were studied for different clusters emitted in the considered region. In section II we describe the main features of the model chosen for the study. The extension of the model to study the cluster decays in the new region is also described in this section. The results of the present study is discussed in Section III.

## Details of calculation

To study the cluster radioactivity in the considered nuclei we have used the Cubic Potential Model which

uses Cubic plus Yukawa plus Exponential potential [8,10,11] for fixing the top of the barrier correctly along with the Coulomb potential for the outer region after separation and a third order polynomial for the overlapping region.

If  $Q$  is the energy released in the reaction, its value is given by

$$Q = [M(Z, A) - M(Z_1, A_1) - M(Z_2, A_2)] \times 931.501 \text{ MeV} \quad (1)$$

$$\text{where } Z = Z_1 + Z_2 \text{ and } A = A_1 + A_2$$

If the  $Q$ -value of the reaction is taken as the origin, then for the post scission region the potential as a function of  $r$  (which is the distance of mass centres of the fragments) is given by,

$$V(r) = Z_1 Z_2 \frac{e^2}{r} + V_n(r) - Q ; \quad r \geq r_{12} \quad (2)$$

where  $V_n(r)$  is the nuclear interaction energy and is.,

$$= -D \left( F + \frac{S}{a} \right) \left( \frac{r_{12}}{r} \right) e^{-\frac{S}{a}} ; \quad S \geq 0$$

$$S = r - r_{12}$$

and

$$r_{12} = r_1 + r_2$$

where  $S$  is the distance between the inner equivalent sharp surfaces of two nuclei.  $r_1$  and  $r_2$  are the sharp surface radii and  $r$  is the distance of mass centres of the fragments.

The depth constant  $D$  can be given as

$$D = \frac{(4a^3 g(\frac{r_1}{a}) g(\frac{r_2}{a}) e^{-\frac{r_{12}}{a}} C_s)}{(r_0^2 r_{12})} \quad (3)$$

$$\text{Where } g(x) = x \cosh x - \sinh x$$

For the case of two separated nuclei,

$$C_s = [C_s(1) C_s(2)]^{\frac{1}{2}}$$

The dependence of the effective surface energy constant  $C_s$  upon the relative neutron and proton excess is taken to be,

$$C_s = a_s (1 - K_s I^2)$$

$$I = \frac{(N - Z)}{A}$$

$$\text{then } C_s(1) = \left( \frac{a_1}{a_1 - a_2} \right)$$

$$\text{and } C_s(2) = \left( \frac{a_2}{a_1 - a_2} \right)$$

where

$a_s$  - surface energy constant

$K_s$  - surface asymmetry constant

$a_1$  and  $a_2$  are the ranges.

The constant F is given by

$$F = 4 + \left( \frac{r_{12}}{a} \right) - \left[ \frac{f(r_1/a)}{g(r_1/a)} \right] - \left[ \frac{f(r_2/a)}{g(r_2/a)} \right] \quad (4)$$

$$\text{and } f(x) = x^2 \sinh x$$

Between the two separated fragments, there should be an attractive nuclear interaction energy besides the Coulomb repulsion. The range of that interaction force should extend beyond the equivalent sharp radius by roughly the range of the nucleon interaction. The nuclear interaction energy of two overlapping spheres of radii  $r_1$  and  $r_2$  and centre of mass distance  $r \geq (r_1 + r_2)$  is given by

$$E_{\text{int}}(r) = -G \frac{e^{-\frac{r}{a}}}{r} \quad (5)$$

where

$$G = 4a \left( \frac{a}{r_0} \right)^2 C_s g \left( \frac{r_1}{a} \right) g \left( \frac{r_2}{a} \right) \quad (6)$$

The values chosen for the parameters are

$$r_0 = 1.16 \text{ fm},$$

$$a = 0.68 \text{ fm},$$

$$a_s = 21.13 \text{ MeV}$$

$$\text{and } K_s = 2.3$$

The shape of the barrier in the overlapping region, which connects the ground state and the contact point is approximated by a third order polynomial suggested by Nix [12]. The barrier should be parabolic near its top and it should have a local minimum corresponding to the ground state equilibrium configuration. The cubic shape is the simplest form, which satisfies these two physical requirements. So, for the overlapping region, we approximate the potential by a third order polynomial in  $r$  having the form

$$V(r) = -E_v + [V(r_{12}) + E_v] \left\{ s_1 \left[ \frac{(r - r_i)}{(r_{12} - r_i)} \right]^2 - s_2 \left[ \frac{(r - r_i)}{(r_{12} - r_i)} \right]^3 \right\} \quad (7)$$

Where  $r_i$  is the distance between the centres of mass of two portions of a spherical parent cut by a planar

section into two pieces with volume asymmetry. Expression for  $r_i$  of a spherical parent nucleus is given by

$$r_i = \frac{3}{4} \left[ \frac{h_1^2}{R_0 + h_1} + \frac{h_2^2}{R_0 + h_2} \right] \quad (8)$$

where  $h_1$  and  $h_2$  are the heights of heavy and light spherical segments respectively.

For the symmetric case  $h_1 + h_2 = 2R_0$ , this reduces to

$$r_i = \frac{3}{4} R_0.$$

The constants  $s_1$  and  $s_2$  are determined by requiring that the value of the potential  $V(r)$  and its first derivative be continuous at the contact point.

In order to accomplish the conservation of energy, the consistent procedure is followed to fit the cubic part of the barrier not to zero at  $r = r_i$  but as  $-E_v$ . The zero point vibration energy [13] is given by,

$$E_v = \frac{\pi h (2Q/\mu)^{\frac{1}{2}}}{2(C_1 + C_2)} \quad (9)$$

$$\text{Here, } C_i = 1.18 A_i^{1/3} - 0.48 \text{ fm} \quad (i=1,2)$$

$C_1$  and  $C_2$  are central radii of the fragments.

where  $\mu = M \frac{A_1 A_2}{A_1 + A_2}$  with  $M$  being the mass of

the nucleon like in fission the half-life of the meta-stable system [14] can be calculated by using the relation

$$T = \frac{\hbar \ln 2}{4\pi E_v} (1 + \exp K) \quad (10)$$

Expressing the half life time in seconds,  $E_v$  and  $\hbar$  in MeV, for the lifetime we have

$$T = \frac{1.433 \times 10^{-21}}{E_v} (1 + \exp K) \quad (11)$$

According to WKB theory, the probability of penetration through the barrier is expressed as

$$P = \frac{1}{[1 + \exp(K)]} \quad (12)$$

The action integral  $K$  is given by,

$$K = K_L + K_R \quad (13)$$

$$\text{Where } K_L = \frac{2}{\hbar} \int_{r_a}^{r_2} [2B_r(r)V(r)]^{\frac{1}{2}} dr \quad (14)$$

$$\text{and } K_R = \frac{2}{\hbar} \int_{r_{12}}^{r_b} [2B_r(r)V(r)]^{\frac{1}{2}} dr \quad (15)$$

The limits of integration  $r_a$  and  $r_b$  are two appropriate zeros of the integral which are found by Newton-Raphson method.

The nuclear inertia  $B_r(r)$  (effective mass) is associated with the motion in the fission direction which is taken to be deformation dependent. Moller et al. [15] proposed a relation for nuclear inertia of symmetric fission in the new valley of the semi empirical model as,

$$B_r(r) - \mu = f(r, r_{12})k(B_r^{irr} - \mu) \quad (16)$$

$k$  is the semi-empirical constant and its value is taken to =16,

$$f(r, r_{12}) = \begin{cases} \left[ \frac{r_{12} - r}{r_{12} - r_i} \right]^4 & \text{for } r \leq r_{12} \\ 0 & \text{for } r \geq r_{12} \end{cases}$$

and  $\mu$  is the reduced ass.

The inertia has the property that it is equal to the old inertia at the spherical shape and is equal to zero at the scission point. They have concluded that, there is much more inertia associated with fusion in the new valley than in the old valley because in former, a single particle level structure in the fissioning system approaches the structure of the final system very easily in the fission process.

$B^{irr}$  is the inertia corresponding to hydrodynamical irrotational flow whose numerical results for asymmetric fission are approximated by Moller and Nix as

$$B^{irr} - \mu = \frac{17}{15} \mu \exp \left[ -\frac{128}{51} \left( \frac{r - r_i}{\alpha_0} \right) \right] \quad (17)$$

where  $\alpha_0$  is the unequal axis of the spherical parent nucleus.

This method is applied to calculate the logarithm of lifetime in seconds for the spontaneous emission of heavier fragments from the considered nuclei.

The Geiger – Nuttal plot is drawn using the equation

$$\log_{10} T_{1/2} = X Q^{-1/2} + Y \quad (18)$$

## Results and Discussion

Cluster Radioactivity is a process by which nuclei equal and heavier than  $\alpha$  particles emitted spontaneously. The clusters usually emitted in this process are the  $\alpha$  particle, Carbon, Oxygen, Neon, Magnesium, and Silicon etc. When the mass of the cluster becomes comparable with the mass of the daughter, symmetric fission takes place. Thus the cluster radioactivity is an intermediate process between the well-known alpha decay and the spontaneous fission. In early years such cluster radioactivity was found mostly in actinide nuclei like Radium, Uranium etc. Very recently it has been predicted that such decays are possible in a new region around  $^{114}\text{Ba}$ . There has been an exciting experimental detection of cluster emissions from nuclei in the region with  $Z=52$  to  $Z=62$ , which are attracting a lot of attention recently.

To study the phenomenon of cluster radioactivity there are various theoretical models in vogue. The existing models generally fall into two categories: the Unified Fission Model (UFM) and Preformed Cluster Model (PCM). In the PCM clusters are assumed to be pre-born in a parent nucleus before they could penetrate the potential barrier with a given  $Q$  value. The UFM considers cluster radioactivity simply as a barrier penetration phenomenon in between the fission and  $\alpha$  decay without worrying about the cluster being or not being pre-formed in the parent nucleus. The cubic potential model is a Unified Fission Model for the study of exotic nuclear decay, which has Cubic plus Yukawa plus Exponential potential for the post scission region. This model has been quoted as a realistic model due to the reason that, in this model a realistic potential is included and also it does not require any redistribution of charge when the parent decays into its products. It is also to be noted that this model fits the cluster decay rates well into all regions without the requirement of additional parameters.

In this work we studied the Exotic Cluster Decay using CYEM for various cluster emissions from the elements with  $Z=52$  to  $Z=62$  and we compared the results with ASAFM. The possible cluster emissions chosen for the study are  $^{16}\text{O}$  and  $^{24}\text{Mg}$ .

In the region considered  $Z=52$  to  $Z=62$  the nuclei of elements like Barium ( $Z=56$ ) in s-Block, Cerium ( $Z=58$ ), Neodymium ( $Z=60$ ) and Samarium ( $Z=62$ ) in 4f - block are taken for the study.

In the first phase of this work we have generated the Exotic Cluster Decay lifetimes for various Cluster emissions from the above said elements using cubic potential model (CYEM). The obtained lifetime values are tabulated and then compared with the values obtained by Poenaru et al. who used the Analytical Super Asymmetric Fission Model (ASAFM) [1]. Our study reveals that the lifetime values calculated using cubic potential model (CYEM) are in agreement with the values obtained by ASAFM. The lowest  $\log_{10} T_{1/2}$  value of certain emissions stresses the fact that these transitions are most favourable for measurements.

The Geiger-Nuttal plots for various cluster emissions from the considered nuclei are calculated and the sample results are presented in figures 3.1 to 3.4. These plots are found to be linear with different slopes and intercepts. This indicates that the inclusion of Yukawa and exponential potential does not produce significant deviation to the linear nature of these Geiger-Nuttal plots which agrees with the earlier calculations. The rate of radioactive decay is found to be inversely proportional to neutron excess in the present nucleus. From the above discussions of the result obtained for various cluster emission possibilities, it is clear that the presence of neutron excess will slow down the exotic decay process.

**Table 1** Logarithm of half-lives for  $^{16}\text{O}$  emissions from Barium isotopes.

Parent	Cluster	Daughter	Q MeV	$\log_{10} T_{1/2}(\text{s})$	
				ASAFM Ref.[1]	CYEM Calculated
$^{114}\text{Ba}_{56}$	$^{16}\text{O}_8$	$^{98}\text{Cd}_{48}$	27.08	13.60	12.58
			26.53	14.70	13.83
			26.95	13.90	12.88
			27.02	13.70	12.72
			25.96	16.00	15.16
			27.74	12.30	12.13
$^{115}\text{Ba}_{56}$	$^{16}\text{O}_8$	$^{99}\text{Cd}_{48}$	25.83	17.80	15.36
			25.41	14.70	16.37
			25.42	19.10	16.87
			25.85	17.70	15.63
			25.00	19.60	17.32
			26.36	16.60	15.51
$^{116}\text{Ba}_{56}$	$^{16}\text{O}_8$	$^{100}\text{Cd}_{48}$	24.78	18.50	17.84
			24.38	19.40	18.86
			24.02	20.30	19.81
			24.81	18.40	17.76
			26.33	15.00	14.07

**Table 2** Logarithm of half-lives for  $^{24}\text{Mg}$  emissions from Cerium isotopes.

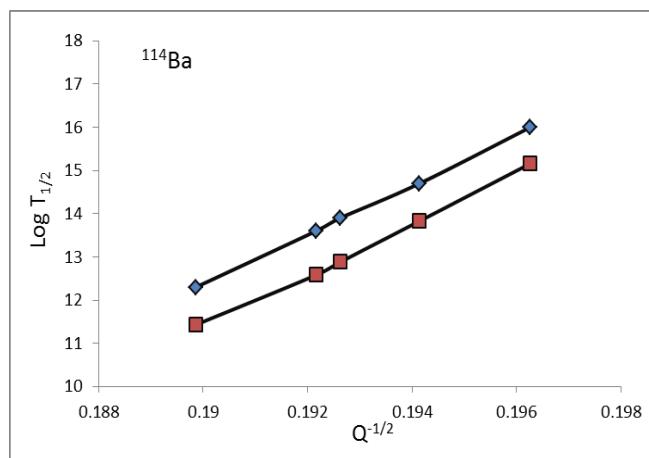
Parent	Cluster	Daughter	Q MeV	$\log_{10} T_{1/2}(\text{s})$	
				ASAFM Ref. [1]	CYEM Calculated
$^{120}\text{Ce}_{58}$	$^{24}\text{Mg}_{12}$	$^{96}\text{Pa}_{46}$	41.42	23.60	21.91
			38.71	28.80	27.63
			40.88	24.60	23.01
			42.25	22.10	20.26
			40.26	25.70	24.30
			40.56	25.20	23.67
$^{121}\text{Ce}_{58}$	$^{24}\text{Mg}_{12}$	$^{97}\text{Pa}_{46}$	39.91	27.70	24.89
			39.23	29.00	26.36
			39.36	28.80	26.07
			38.87	29.80	27.14
			39.26	29.00	26.29

**Table 3** Logarithm of half-lives for  $^{24}\text{Mg}$  emissions from Neodymium isotopes.

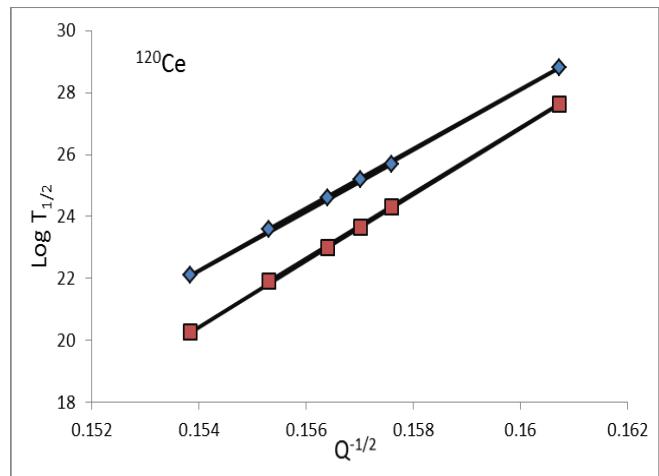
Parent	Cluster	Daughter	Q MeV	$\log_{10} T_{1/2}(\text{s})$	
				ASAFM Ref. [1]	CYEM Calculated
$^{122}\text{Nd}_{60}$	$^{24}\text{Mg}_{12}$	$^{98}\text{Cd}_{48}$	46.31	19.00	17.30
			44.21	22.60	21.20
			47.00	17.90	16.07
			45.90	19.70	18.05
$^{123}\text{Nd}_{60}$	$^{24}\text{Mg}_{12}$	$^{99}\text{Cd}_{48}$	46.45	20.10	16.89
			44.68	23.00	20.15
			42.27	27.30	24.88
			44.94	22.60	19.66
			44.18	23.90	21.11
			45.39	21.80	18.83
$^{124}\text{Nd}_{60}$	$^{24}\text{Mg}_{12}$	$^{100}\text{Cd}_{48}$	44.92	21.10	19.55
			43.09	24.40	23.09
			40.98	28.30	27.42
			43.64	23.40	22.00
			44.65	21.60	20.06
			42.54	25.40	24.19
			43.85	23.00	21.59

**Table 4** Logarithm of half-lives for  $^{24}\text{Mg}$  emissions from Samarium isotopes.

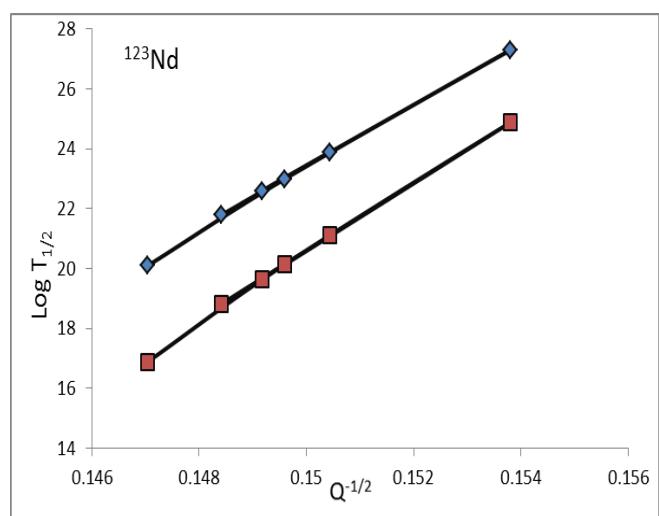
Parent	Cluster	Daughter	Q MeV	Log <sub>10</sub> T <sub>1/2</sub> (s)	
				ASAFM Ref. [1]	CYEM Calculated
$^{127}\text{Sm}_{62}$	$^{24}\text{Mg}_{12}$	$^{103}\text{Sn}_{50}$	46.99	22.80	20.14
			44.36	27.30	25.14
			47.02	22.70	20.09
			46.78	23.10	20.53
$^{128}\text{Sm}_{62}$	$^{24}\text{Mg}_{12}$	$^{104}\text{Sn}_{50}$	45.71	23.40	22.38
			43.34	27.70	27.05
			45.96	23.00	21.91
			45.54	23.70	22.70
$^{129}\text{Sm}_{62}$	$^{24}\text{Mg}_{12}$	$^{105}\text{Sn}_{50}$	47.05	22.50	19.73
			44.32	27.20	24.94
			43.92	28.00	25.74
			49.09	19.20	16.09
$^{130}\text{Sm}_{62}$	$^{24}\text{Mg}_{12}$	$^{106}\text{Sn}_{50}$	44.25	27.40	25.08
			45.97	22.80	21.60
			43.15	27.90	27.17
			43.22	27.70	27.02
			48.11	19.20	17.67



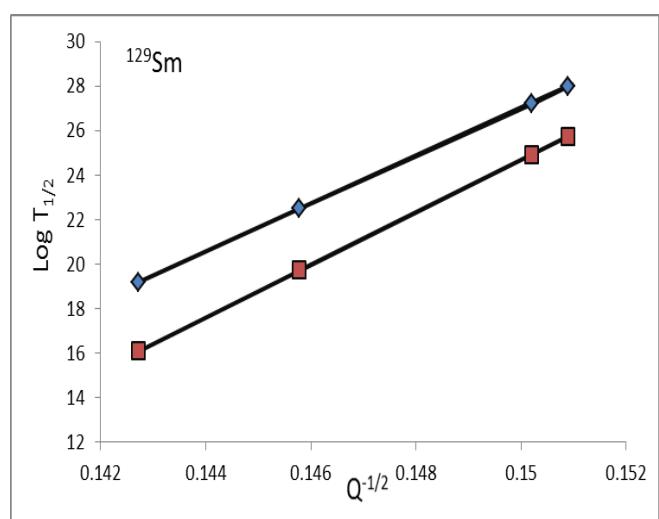
**Fig 1** Geiger–Nuttal Plots for  $^{16}\text{O}$  emission from  $^{114}\text{Ba}$



**Fig 2** Geiger–Nuttal plots for  $^{24}\text{Mg}$  emission from  $^{120}\text{Ce}$



**Fig 3** Geiger–Nuttal plots for  $^{24}\text{Mg}$  emission from  $^{123}\text{Nd}$



**Fig 4** Geiger–Nuttal plots for  $^{24}\text{Mg}$  emission from  $^{129}\text{Sm}$

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