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RESEARCH ARTICLE

On contra $sarps$-continuous functions in topological spaces

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Abstract

In 1970, Levine introduced generalized closed sets in topological spaces in order to extend many of the important properties of closed sets to a large family. In the recent past, there has been considerable interest in the study of various forms of generalized closed sets. The authors introduced $sarps$-closed sets in topological spaces. In this, we introduce a new class of function called contra $sarps$-continuous functions by using $sarps$-closed sets and characterize their basic properties. Further the relationship between this new class with other classes of existing contra continuous functions are established. Also we define contra $sarps$-irresolute, perfectly contra $sarps$-irresolute and almost contra $sarps$-continuous functions and we have given the relationship of these three functions with contra $sarps$-continuous functions.

Keywords

Contra $sarps$-continuous

Contra $sarps$-irresolute

Perfectly contra $sarps$-irresolute

Almost contra $sarps$-continuous

1. Introduction

In 1968, M. K. Singal and A. R. Singal [1] introduced almost continuous mappings. In 1986, T. Noiri introduced the concept of perfectly continuous. In 1996, J.Dontchev [2] introduced the notion of contra continuity. In 1999, J. Dontchev and T. Noiri [3] introduced new class of functions, called contra semi-continuous functions. The authors introduced $sarps$-closed sets in topological spaces. The purpose of this paper is to introduce a new class of functions, namely contra $sarps$-continuous functions in topological spaces.

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2. Preliminaries

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X \ A denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1: A subset A of a space X is called

(i) semi-open [19] if A $\subseteq$ *cl int*A and semi-closed if *int cl*A $\subseteq$ A.

(ii) α-open [24] if A $\subseteq$ *int cl int*A and α-closed if *cl int cl*A $\subseteq$ A.

(iii) π-open [4] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The semi-closure (resp. pre-closure, resp. semi-pre-closure, resp. α-closure, resp. b-closure) of a subset A of X is the intersection of all semi-closed (resp. pre-closed, resp. semi-pre-closed, resp. α-closed, resp. b-closed) sets containing A and is denoted by *scl*A (resp. *pcl*A, resp. *spcl*A, resp. αclA, resp. *bcl*A).

Definition 2.2: A subset A of a space X is called g-closed [20] (resp. rg-closed [26], resp. αg-closed [21], resp. gs-closed [5], resp. gp-closed [22], resp. gpr-closed [13], resp. gsp-closed [8], resp. πg-closed [11], resp. πgp-closed [27], resp. $\pi g\alpha$-closed [17], resp. πgb-closed [4], resp. rwg-closed [35], resp. gb-closed [1], resp. g^*p-closed [36], resp. rgb-closed [23], resp. *g-closed [37]) if *cl*A $\subseteq$ U (resp. *cl*A $\subseteq$ U, resp. $\alpha clA \subseteq$ U, resp. *scl*A $\subseteq$ U, resp. *pcl*A $\subseteq$ U, resp. *pcl*A $\subseteq$ U, resp. *spcl*A $\subseteq$ U, resp. *cl*A $\subseteq$ U, resp. *pcl*A $\subseteq$ U, resp. $\alpha clA \subseteq$ U, resp. *bcl*A $\subseteq$ U, resp. *cl int*A $\subseteq$ U, resp. *bcl*A $\subseteq$ U, resp. *pcl*A $\subseteq$ U, resp. *bcl*A $\subseteq$ U, resp. *cl*A $\subseteq$ U) whenever A $\subseteq$ U and U is open (resp. regular open, resp. open, resp. open, resp. open, resp. regular open, resp. open, resp. π-open, resp. π-open, resp. π-open, resp. π-open, resp. regular open, resp. open, resp. g-open, resp. regular open, resp. $\hat{g}$-open.).

Definition 2.3 [33]

A subset A of a space X is called semi α-regular pre-semi closed (briefly $s\alpha rps$-closed) if *scl*A $\subseteq$ U whenever A $\subseteq$ U and U is αrps-open.

The complements of the above mentioned closed sets are their respective open sets. For example, a subset B of a space X is generalized open (briefly g-open) if X \ B is g-closed.

Definition 2.4

(i) A function f: $(X,\tau)\to(Y,\sigma)$ is called [6] if $f^{-1}(V)$ is closed in (X,τ) for every closed subset V of (Y,σ).

(ii) A function f: $(X,\tau)\to(Y,\sigma)$ is called perfectly-continuous [25] if $f^{-1}(V)$ is clopen in (X,τ) for every closed subset V of (Y,σ).

(iii) A function f: $(X,\tau)\to(Y,\sigma)$ is called regular set connected [14] if $f^{-1}(V)$ is clopen in (X,τ) for every regular closed subset V of (Y,σ).

(iv) almost continuous[31] if $f^{-1}(V)$ is closed in (X,τ) for every regular closed subset V of (Y,σ).

Definition 2.5[29]

(i) A function f: $(X,\tau)\to(Y,\sigma)$ is called $s\alpha rps$-continuous if $f^{-1}(V)$ is $s\alpha rps$-closed in (X,τ) for every closed subset V of (Y,σ).

(ii) A function f: $(X,\tau)\to(Y,\sigma)$ is called $s\alpha rps$-irresolute if $f^{-1}(V)$ is $s\alpha rps$-closed in (X,τ) for every $s\alpha rps$-closed subset V of (Y,σ).

(iii) A function f: $(X,\tau)\to(Y,\sigma)$ is called almost $s\alpha rps$-continuous if $f^{-1}(V)$ is $s\alpha rps$-closed in (X,τ) for every regular closed subset V of (Y,σ).

Definition 2.6

A function f: $(X,\tau)\to(Y,\sigma)$ is called $s\alpha rps$-closed (resp. $s\alpha rps$-open) if for every closed (resp. open) set U of (X,τ), the set f(U) is $s\alpha rps$-closed (resp. $s\alpha rps$-open) in (Y,σ).

Definition 2.7

A function f: $(X,\tau)\to(Y,\sigma)$ is called contra continuous [9] (resp. contra semi-continuous [10], resp. contra π-continuous [12], resp. contra α-continuous [16], resp. contra g-continuous [6], resp. contra rg-continuous [34], resp. contra αg-continuous [3], resp. contra gs-continuous [14], resp. contra gp-continuous [7], resp. contra gpr-continuous [14], resp. contra gsp-continuous [3], resp. contra πg-continuous [12], resp. contra πgp-continuous [7], resp. contra πgb-continuous [32], resp. contra rwg-continuous [34], resp. contra gb-continuous [2], resp. contra g^*p-continuous [3], resp. contra $\pi g\alpha$-continuous [17], resp. contra *g-continuous [34], resp. contra rgb-continuous [30]) if $f^{-1}(V)$ is closed (resp. semi-closed, resp. π-closed, resp. α-closed, resp. g-closed, resp. rg-closed, resp. αg-closed, resp. gs-closed, resp. gp-closed, resp. gpr-closed, resp. gsp-closed, resp. πg-closed, resp. πgp-closed, resp. πgb-closed, resp. rwg-closed, resp. gb-closed, resp. g^*p-closed, resp. $\pi g\alpha$-closed, resp. *g-closed, resp. rgb-closed) in (X,τ) for every open subset V of (Y,σ).

Lemma 2.8

Every closed set is $s\alpha rps$-closed.

Definition 2.9 [18]

A space X is called locally indiscrete if every open subset of X is closed.

Contra $S\alpha RPS$-continuous functions

Definition 3.1

A function f: $(X,\tau)\to(Y,\sigma)$ is called contra $s\alpha rps$-continuous if $f^{-1}(V)$ is $s\alpha rps$- closed in (X,τ) for every open subset V of (Y, σ).

Proposition 3.2

If A function f: (X, τ) $\rightarrow$ (Y, σ) from a topological space X into a topological space Y is contra-continuous, then it is contra $s\alpha rps$-continuous.

Proof

Assume that the function f: (X, τ) $\rightarrow$ (Y, σ) is contra-continuous. Let V be an open subset of (Y, σ). Since f is contra-continuous, f^{-1}(V) is closed in (X, τ). By Lemma 2.8, f^{-1} (V) is $s\alpha rps$-closed in (X, τ). Hence f is contra $s\alpha rps$-continuous.

Converse of the above Proposition need not be true as shown in the following example.

Example 3.3

Let X = {a,b,c} with topology τ ={ϕ,{a},{b,c},X} and Y = {p,q} with topology σ ={ϕ,{p},Y}. Let f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = f(c) = q, f(b) = p. Then f is contra $s\alpha rps$-continuous, but not contra-continuous.

Proposition 3.4

Let f: (X, τ) $\rightarrow$ (Y, σ) be a function. Then

(i) if f is contra semi-continuous, then f is contra $s\alpha rps$-continuous.

(ii) if f is contra -continuous, then f is contra -continuous.

(iii) if f is contra -continuous, then f is contra -continuous.

Proof

Suppose f is contra semi-continuous (resp. contra π-continuous, resp. contra α-continuous). Let V be an open subset of (Y, σ). Since f is contra semi-continuous (resp. contra π-continuous, resp. contra α-continuous), f^{-1}(V) is semi-closed (resp. π-closed, resp. α-closed) in (X, τ). Using Proposition 3.2 of [33], f^{-1}(V) is $s\alpha rps$-closed in (X, τ). Then by using Definition 3.1, f is contra $s\alpha rps$-continuous. This proves (i), (ii) and (iii).

The reverse implications need not be true as shown in the Example 3.5.

Example 3.5

Let X = Y={a,b,c} with topologies τ ={ϕ,{a},{b,c},X} and σ ={ϕ,{a},Y} on X and Y respectively. Let the function f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = c, f(b) = a, f(c) = b.

Then f is contra $s\alpha rps$-continuous, but not contra semi-continuous, not contra π-continuous, not contra α-continuous.

Proposition 3.6

Let f: (X, τ) $\rightarrow$ (Y, σ) be a function. Then

(i) if f is contra $s\alpha rps$-continuous, then f is contra gs-continuous.

(ii) if f is contra $s\alpha rps$-continuous, then f is contra rgb-continuous.

(iii) if f is contra $s\alpha rps$-continuous, then f is contra πgb-continuous.

(iv) if f is contra $s\alpha rps$-continuous, then f is contra gb-continuous.

(v) if f is contra $s\alpha rps$-continuous, then f is contra gsp-continuous.

Proof

Suppose f is contra $s\alpha rps$-continuous. Let V be an open subset of (Y, σ). Since f is contra $s\alpha rps$-continuous, f^{-1}(V) is $s\alpha rps$-closed in (X, τ). Then by using Proposition 3.4 of [33], f^{-1}(V) is gs-closed (resp. rgb -closed, resp. πgb-closed) in (X, τ). Therefore f is contra gs-continuous (resp. contra rgb-continuous, resp. contra πgb-continuous). This proves (i), (ii) and (iii). Since gs-closed $\Rightarrow$ gb-closed $\Rightarrow$ gsp-closed, the proof for (iv) and (v) follows from (i).

The reverse implications need not be true as shown in the Example 3.7.

Example 3.7

Let X = Y = {a,b,c,d} with topologies τ ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},X} and σ ={ϕ,{a,c},Y} on X and Y respectively.

Let the function f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = a, f(c) = b, f(d) = c. Then f is contra gs-continuous, contra rgb-continuous, contra πgb-continuous, contra gb-continuous, contra gsp-continuous, but not contra $s\alpha rps$-continuous.

The concept contra $s\alpha rps$-continuous is independent from the concepts contra αg-continuous, contra πg-continuous, contra gp-continuous, contra πgp-continuous, contra $\pi g\alpha$-continuous, contra g^*p-continuous, contra rg-continuous, contra g-continuous, contra gpr-continuous, contra rwg-continuous, contra *g-continuous as shown in the following examples.

Example 3.8

From Example 3.7, f^{-1}({a,c}) = {b,d} is αg-closed, πg-closed, gp-closed, πgp-closed, $\pi g\alpha$-closed, g^*p-closed in (X, τ). Hence f is contra αg-continuous, contra πg-continuous, contra gp-continuous, contra πgp-continuous, contra $\pi g\alpha$-continuous, contra g^*p-continuous, but not contra $s\alpha rps$-continuous.

Example 3.9

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},X} and σ={ϕ,{b},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = a, f(c) = b, f(d) = c. Then f is contra $s\alpha rps$-continuous, but not contra αg-continuous, not contra πg-continuous, not contra gp-continuous, not contra πgp-continuous, not contra g^*p-continuous.

Example 3.10

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{a,b},X} and σ={ϕ,{d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = d, f(c) = a, f(d) = c. Then f is contra $s\alpha rps$-continuous, but not contra g-continuous.

Example 3.11

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{a,b},X} and σ={ϕ,{b,c},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = c, f(b) = d, f(c) = b, f(d) = a. Then f is contra g-continuous, but not contra $s\alpha rps$-continuous.

Example 3.12

Let X = Y = {a,b,c} with topologies τ={ϕ,{a},{b},{a,b},X} and σ={ϕ,{c},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = c, f(b) = a, f(c) = b. Then f is contra $s\alpha rps$-continuous, but not contra gpr-continuous.

Example 3.13

Let X = Y = {a,b,c} with topologies τ={ϕ,{a},{b},{a,b},X} and σ={ϕ,{b,c},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = a. Then f is contra gpr-continuous, but not contra $s\alpha rps$-continuous.

Example 3.14

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{b},{a,b},{a,b,c},X} and σ={ϕ,{c},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Then f is contra $s\alpha rps$-continuous, but not contra rwg-continuous and contra *g-continuous.

Example 3.15

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{b},{a,b},{a,b,c},X} and σ={ϕ,{c,d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = c, f(b) = d, f(c) = a, f(d) = c. Then f is contra rwg-continuous and contra *g-continuous, but not contra $s\alpha rps$-continuous.

Example 3.16

Let X = Y = {a,b,c} with topologies τ={ϕ,{a},{b},{a,b},{a,b,c},{a,b,d},X} and σ={ϕ,{b,c,d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Then f is contra rg-continuous, but not contra $s\alpha rps$-continuous.

Example 3.17

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{b},{a,b},{a,b,c},{a,b,d},X} and σ={ϕ,{d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = d, f(b) = c, f(c) = b, f(d) = a. Then f is contra $s\alpha rps$-continuous, but not contra rg-continuous.

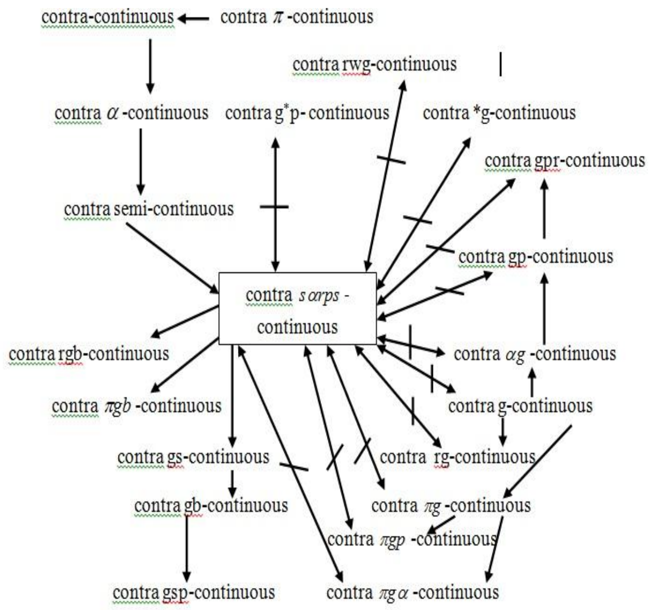
The concept of $s\alpha rps$-continuity and contra $s\alpha rps$-continuity are independent of each other as shown in the Examples 3.18 and 3.19.

Example 3.18

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},X} and σ={ϕ,{a,c,d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = b, f(b) = a, f(c) = b, f(d) = c. Then f is $s\alpha rps$-continuous, but not contra $s\alpha rps$-continuous.

Example 3.19

Let X = Y = {a,b,c,d} with topologies τ={ϕ,{a},{a,b},X} and σ={ϕ,{a,d},Y} on X and Y respectively. Let the function f: (X,τ)$\to$(Y,σ) be defined as f(a) = c, f(b) = d, f(c) = b, f(d) = a. Then f is contra $s\alpha rps$-continuous, but not $s\alpha rps$-continuous. Thus the above discussions lead to the following diagram. In this diagram, "A$\to$B" means A implies B but not conversely and "A $\not\leftrightarrow$ B" means A and B are independent of each other.



Definition 3.20

A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called contra $s\alpha rps$-irresolute if $f^{-1}(V)$ is $s\alpha rps$- closed in (X,τ) for every $s\alpha rps$-open subset V of (Y,σ). The concepts of $s\alpha rps$-irresolute and contra $s\alpha rps$-irresolute are independent of each other as shown in the Examples 3.21 and 3.22.

Example 3.21

Let X = Y = {a,b,c} with topologies $\tau=\{\phi,\{b,c\},X\}$ and $\sigma=\{\phi,\{a\},\{a,c\},Y\}$ on X and Y respectively. Let the function f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = a, f(b) = b, f(c) = c. Then f is contra $s\alpha rps$-irresolute, but not $s\alpha rps$-irresolute.

Example 3.22

Let X = Y = {a,b,c} with topologies $\tau=\{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma=\{\phi,\{b,c\},Y\}$ on X and Y respectively. Let the function f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = a. Then f is $s\alpha rps$-irresolute, but not contra $s\alpha rps$-irresolute.

Theorem 3.23

Every contra $s\alpha rps$-irresolute function is contra $s\alpha rps$-continuous.

Proof

Suppose f: $(X,\tau)\rightarrow(Y,\sigma)$ is contra $s\alpha rps$-irresolute. Let V be any open subset of (Y,σ). Since every open set is semi-open and by using Proposition 3.2 of [28], V is $s\alpha rps$-open in (Y,σ). Since f is contra $s\alpha rps$-irresolute, $f^{-1}(V)$ is $s\alpha rps$-closed in (X,τ). Hence f is contra $s\alpha rps$-continuous. Converse of the above theorem need not be true as seen in the following example.

Example 3.24

Let X = Y = {a,b,c} with topologies $\tau=\{\phi,\{a\},\{a,c\},X\}$ and $\sigma=\{\phi,\{c\},\{b,c\},Y\}$ on X and Y respectively. Let the function f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = a, f(b) = b, f(c) = c. Then f is contra $s\alpha rps$-continuous, but not contra $s\alpha rps$-irresolute.

Theorem 3.25

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be two functions. Let h = $g\circ f$. Then

(i) h is contra $s\alpha rps$-continuous if f is contra $s\alpha rps$-continuous and g is continuous.

(ii) h is contra $s\alpha rps$-continuous if f is $s\alpha rps$-irresolute and g is contra-continuous.

(iii) h is contra $s\alpha rps$-continuous if f is $s\alpha rps$-irresolute and g is contra $s\alpha rps$-continuous.

(iv) h is $s\alpha rps$-continuous and contra $s\alpha rps$-continuous if f is contra $s\alpha rps$-continuous and g is perfectly-continuous.

(v) h is $s\alpha rps$-continuous if f is contra $s\alpha rps$-continuous and g is contra-continuous.

Proof

(i) Let V be open in (Z,μ). Since g is continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra $s\alpha rps$-continuous, $f^{-1}(g^{-1}(V))$ is $s\alpha rps$-closed in (X,τ). That is, $(g\circ f)^{-1}(V)$ is $s\alpha rps$-closed in (X,τ). This proves (i).

(ii) Let V be open in (Z,μ). Since g is contra-continuous, $g^{-1}(V)$ is closed in (Y,σ). By Lemma 2.8, $g^{-1}(V)$ is $s\alpha rps$-closed in (Y,σ). Since f is $s\alpha rps$-irresolute, $f^{-1}(g^{-1}(V))$ is $s\alpha rps$-closed in (X,τ). That is, $(g\circ f)^{-1}(V)$ is $s\alpha rps$-closed in (X,τ). This proves (ii).

(iii) Let V be open in (Z,μ). Since g is contra $s\alpha rps$-continuous, $g^{-1}(V)$ is $s\alpha rps$-closed in (Y,σ). Since f is $s\alpha rps$-irresolute, $f^{-1}(g^{-1}(V))$ is $s\alpha rps$-closed in (X,τ). That is, $(g\circ f)^{-1}(V)$ is $s\alpha rps$-closed in (X,τ). This proves (iii).

(iv) Let V be closed in (Z,μ). Since g is perfectly-continuous, $g^{-1}(V)$ is clopen in (Y,σ). Since f is contra $s\alpha rps$-continuous, $f^{-1}(g^{-1}(V))$ is both $s\alpha rps$-closed and $s\alpha rps$-open in (X,τ). That is, $(g\circ f)^{-1}(V)$ is both $s\alpha rps$-closed and $s\alpha rps$-open in (X,τ). This proves (iv).

(v) Let V be closed in (Z, μ). Since g is contra-continuous, $g^{-1}(V)$ is open in (Y, σ). Since f is contra $s\alpha rps$-continuous, $f^{-1}(g^{-1}(V))$ is $s\alpha rps$-closed in (X, τ). That is, $(g \circ f)^{-1}(V)$ is $s\alpha rps$-closed in (X, τ). This proves (v).

Theorem 3.26

Let f: $(X, \tau) \to (Y, \sigma)$ be surjective, $s\alpha rps$-irresolute and $s\alpha rps$-open and g: $(Y, \sigma) \to (Z, \mu)$ be any function. Let every $s\alpha rps$-open set in (X, τ) be open. Then $g \circ f$ is contra $s\alpha rps$-continuous if and only if g is contra $s\alpha rps$-continuous.

Proof

Assume that $g \circ f$ is contra $s\alpha rps$-continuous. Let V be closed in (Z, μ). Since $g \circ f$ is contra $s\alpha rps$-continuous, $(g \circ f)^{-1}(V)$ is $s\alpha rps$-open in (X, τ). By assumption, $(g \circ f)^{-1}(V)$ is open in (X, τ). That is $f^{-1}(g^{-1}(V))$ is open in (X, τ). Since f is $s\alpha rps$-open, $f(f^{-1}(g^{-1}(V)))$ is $s\alpha rps$-open in (Y, σ). That is $g^{-1}(V)$ is $s\alpha rps$-open in (Y, σ). Hence g is contra $s\alpha rps$-continuous. The converse part follows from Theorem 3.25(iii).

Theorem 3.27

The following are equivalent for a function f: $(X, \tau) \to (Y, \sigma)$.
(i) f is contra $s\alpha rps$-continuous.
(ii) The inverse image of each closed set in (Y, σ) is $s\alpha rps$-open in (X, τ).

Proof

(i) $\Rightarrow$ (ii)

Suppose (i) holds. Let V be closed in (Y, σ). Then $Y \setminus V$ is open in (Y, σ). By assumption, $f^{-1}(Y \setminus V)$ is $s\alpha rps$-closed in (X, τ). But $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ which is $s\alpha rps$-closed in (X, τ). Therefore $f^{-1}(V)$ is $s\alpha rps$-open in (X, τ). This proves (i) $\Rightarrow$ (ii).

(ii) $\Rightarrow$ (i)

Let V be open in (Y, σ). Then $Y \setminus V$ is closed in (Y, σ). By assumption, $f^{-1}(Y \setminus V)$ is $s\alpha rps$-open in (X, τ). But $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ which is $s\alpha rps$-open in (X, τ). Therefore $f^{-1}(V)$ is $s\alpha rps$-closed in (X, τ). This proves (ii) $\Rightarrow$ (i).

Definition 3.28

A function f: $(X, \tau) \to (Y, \sigma)$ is called perfectly contra $s\alpha rps$-irresolute if $f^{-1}(V)$ is both $s\alpha rps$-closed and $s\alpha rps$-open in (X, τ) for every $s\alpha rps$-open subset V of (Y, σ).

Theorem 3.29

A function f is perfectly contra $s\alpha rps$-irresolute if and only if f is contra $s\alpha rps$-irresolute and $s\alpha rps$-irresolute.

Proof

Suppose f is perfectly contra $s\alpha rps$-irresolute. Let V be $s\alpha rps$-open in Y. Since f is perfectly contra $s\alpha rps$-irresolute, $f^{-1}(V)$ is both $s\alpha rps$-closed and $s\alpha rps$-open in (X, τ). Hence f is contra $s\alpha rps$-irresolute and $s\alpha rps$-irresolute.
Conversely, suppose f is contra $s\alpha rps$-irresolute and $s\alpha rps$-irresolute. Let V be $s\alpha rps$-open in (Y, σ). Since f is contra $s\alpha rps$-irresolute and $s\alpha rps$-irresolute, $f^{-1}(V)$ is both $s\alpha rps$-closed and $s\alpha rps$-open in (X, τ). Hence f is perfectly contra $s\alpha rps$-irresolute.

Definition 3.30

A function f: $(X, \tau) \to (Y, \sigma)$ is called almost contra $s\alpha rps$-continuous if $f^{-1}(V)$ is $s\alpha rps$-closed in (X, τ) for every regular open subset V of (Y, σ).

Theorem 3.31

Every contra $s\alpha rps$-continuous function is almost contra $s\alpha rps$-continuous.

Proof

Let f: $(X, \tau) \to (Y, \sigma)$ be a function. Suppose f is contra $s\alpha rps$-continuous. Let V be a regular open subset of (Y, σ). Since every regular open set is open, V is open in (Y, σ). Since f is contra $s\alpha rps$-continuous, $f^{-1}(V)$ is $s\alpha rps$-closed in (X, τ). Hence f is almost $s\alpha rps$-continuous.

Conclusion

In this paper, we introduced and investigated the notion of contra $s\alpha rps$-continuous functions by utilizing $s\alpha rps$-closed sets. We obtained fundamental properties of contra $s\alpha rps$-continuous functions and discussed the relationships between contra $s\alpha rps$-continuity and other related functions.

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